AAPT UNITED STATES PHYSICS TEAM
AIP 2002

Solutions to Problems

Any correct solution should be awarded equivalent points. Suggested partial-credit points are presented in square brackets at the right margin. You may further break down the listed points into one point increments. If it is clear they have done an intermediate step, they should get credit for it even if they have not presented it. For example, in 1.a., if a student wrote down mgH = Fh, they should get 9 points credit. Students should not be penalized in a subsequent part for using the wrong answer to a previous part. (No double jeopardy.)

Points

1. a. Using the work-energy theorem $PE_i + KE_i + W = PE_f + KE_f$ [1]

"i" refers to the point of maximum height H and "f" refers to the Earth's surface. At both points the kinetic energy is zero

$KE_i = 0$ $KE_f = 0.$ [2]

Setting the potential energy at the Earth's surface equal to zero

$PE_i = mgH$ $PE_f = 0.$ [2]

The work done by the constant force F $W = \vec{F} \cdot \vec{d}$ [1]

$\vec{F}$ is upward, displacement $\vec{d}$ is downward and has magnitude h. $W = -Fh.$ [2]

Inserting these into the work energy equation $mgH - Fh = 0$ [1]

or $h = \dfrac{mg}{F} H.$ [1]

b. A zero velocity solution is impossible if $h \geq H$. Then [2]

$H \leq \dfrac{mg}{F} H,$ [1]

or $F \leq mg.$ [2]

{*Minus one point if* $<$ *instead of* $\leq$.} {*A check of the equality shows that for* $h = H$, $F = mg$. *So at the highest point its velocity is zero and the net force is zero. It just stays there and will not reach ground with zero velocity. This argument is not required for full credit.*}

c. Using the work-energy theorem $PE_i + KE_i + W = PE_f + KE_f$

"i" refers to the point of maximum height H and "f" refers to the Earth's surface. At both points the kinetic energy is zero

$KE_i = 0$ $KE_f = 0.$ [1]

Here the gravitational potential energy is $PE_i = -\dfrac{GM_E m}{R_E + H}$ $PE_f = -\dfrac{GM_E m}{R_E}.$ [2]

Where M_E is the mass of the Earth. The work done is $W = -Fh$ [2]

(see above). Inserting these into the work energy equation

$$-\frac{GM_E m}{R_E + H} - Fh = -\frac{GM_E m}{R_E} \quad [1]$$

solving for h $$h = \frac{GM_E m}{F}\left(\frac{1}{R_E} - \frac{1}{R_E + H}\right) = \frac{GM_E m}{F}\left(\frac{R_E + H - R_E}{R_E(R_E + H)}\right) = \frac{GM_E mH}{FR_E(R_E + H)}. \quad [1]$$

Answer must be expressed in terms of m, H, F, g, and R_E. Need to eliminate G and M_E.

$$g = \frac{GM_E}{R_E^2} \quad [1]$$

Solving for $$GM_E = gR_E^2. \quad [1]$$

Inserting this into the equation for h

$$h = \frac{(gR_E^2)mH}{FR_E(R_E + H)} = \frac{mgHR_E}{F(R_E + H)} = \frac{mg}{F}H\left(\frac{1}{1 + H/R_E}\right). \quad [1]$$

2. a. When the particle is released, it begins to move with an acceleration which has magnitude

$$a = \frac{F}{m} \quad [2]$$

and is directed toward position E. Since it starts from rest $$A = \tfrac{1}{2}at^2 \quad [2]$$

It reaches E in time t $$t = \sqrt{\frac{2A}{a}} = \sqrt{\frac{2Am}{F}} \quad [1]$$

The particle oscillates back and forth between amplitude A on the right and A on the left. [2]

The time for one complete cycle is four times this $$T = 4t \quad [2]$$

$$T = 4\sqrt{\frac{2Am}{F}} \quad [1]$$

b. The particle oscillates between amplitude A above point E and A' below. $A \neq A'$ [1]

Let t_1 = time it takes the particle to go from amplitude A above to point E and from E up to A.

Let t_2 = time it takes the particle to go from point E to amplitude A' below and from A' up to E.

The period is $$T = 2t_1 + 2t_2 \quad [2]$$

When the particle is above point E, the net force acting on it is

$$ma = \sum F = F + mg \quad \text{downward.} \quad [1]$$

The magnitude of the acceleration is $$a = \frac{F + mg}{m} \quad [1]$$

Starting from rest the particle travels a distance A given by $$A = \tfrac{1}{2}at_1^2 \quad [1]$$

Solving for t_1 $$t_1 = \sqrt{\frac{2A}{a}} = \sqrt{\frac{2Am}{F + mg}} \quad [1]$$

The particle passes through E with velocity $$v_E = at_1 = \left(\frac{F + mg}{m}\right)\sqrt{\frac{2Am}{F + mg}} \quad [1]$$

When the particle is below point E, the net force acting on it is

$$ma = \sum F = F - mg \quad \text{upward} \quad [1]$$

The magnitude of the acceleration is $$a = \frac{F - mg}{m} \quad [1]$$

It reaches maximum downward displacement A' when $v = 0$. [1]

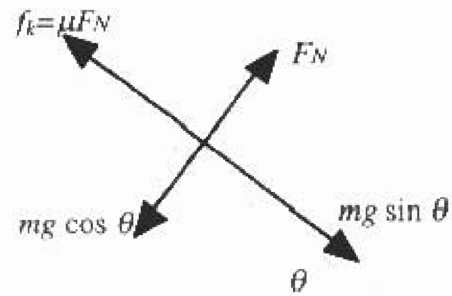
$$v = 0 = v_E - at_2 \quad [1]$$

Solving for t_2 $$t_2 = \frac{v_E}{a} = \left(\frac{F+mg}{m}\right)\sqrt{\frac{2Am}{F+mg}}\left(\frac{m}{F-mg}\right) = \left(\frac{F+mg}{F-mg}\right)\sqrt{\frac{2Am}{F+mg}}$$ [1]

$$T = 2t_1 + 2t_2 = 2\sqrt{\frac{2Am}{F+mg}} + 2\left(\frac{F+mg}{F-mg}\right)\sqrt{\frac{2Am}{F+mg}}$$ [1]

$$T = 2\sqrt{\frac{2Am}{F+mg}}\left(1+\frac{F+mg}{F-mg}\right) = \left(\frac{4F}{F-mg}\right)\sqrt{\frac{2Am}{F+mg}}$$ [1]

3. a. Selecting the x-axis parallel to the inclined plane and y-axis perpendicular to it, the forces acting on the glider are shown as components to the right. Since the glider has no acceleration in the y-direction (perpendicular to the plane),



$$F_N = mg\cos\theta$$ [2]

The acceleration in the x-direction can be obtained from Newton's second law, $$\sum F = ma$$ [2]

$$ma_x = mg\sin\theta - f_k$$ [1]

Subbing in for friction $$f_k = \mu F_N = \mu mg\cos\theta$$ [1]

$$a_x = g\sin\theta - \mu g\cos\theta.$$ [1]

The glider has initial velocity v at time $t = 0$. The displacement of the glider and point LP at any time is given by $$x_{LP} = vt + \tfrac{1}{2}a_x t^2$$ [1]

$$x_{LP} = vt + \tfrac{1}{2}(g\sin\theta - \mu g\cos\theta)t^2$$ [1]

The only force acting on the projectile is gravity. The projectile moves in two dimensions both parallel to the plane and perpendicular to it. The projectile has acceleration components

$a_x = g\sin\theta$ $a_y = -g\cos\theta$ [2]

and initial velocity components $v_x = v$ $v_y = v_o$ [2]

Its x and y displacements are given by $x = vt + \tfrac{1}{2}g\sin\theta t^2$ $y = v_o t - \tfrac{1}{2}g\cos\theta t^2$ [2]

The x-displacement of the projectile relative to point LP is

$$\Delta x = x - x_{LP} = vt + \tfrac{1}{2}g\sin\theta t^2 - \left[vt + \tfrac{1}{2}(g\sin\theta - \mu g\cos\theta)t^2\right] = \tfrac{1}{2}\mu g\cos\theta t^2$$ [2]

When the projectile lands, $$y = 0$$ [1]

$$0 = v_o t - \tfrac{1}{2}g\cos\theta t^2$$

Solving for t and assuming $t \neq 0$ $$t = \frac{2v_o}{g\cos\theta}$$ [1]

$$\Delta x = \tfrac{1}{2}\mu g\cos\theta\left(\frac{2v_o}{g\cos\theta}\right)^2 = \frac{2\mu v_o^{\,2}}{g\cos\theta}$$ [1]

b. If the air is on the system can be assumed to be frictionless, $\mu = 0$. [1]

$$\Delta x = 0$$ [4]

The projectile lands at LP.

4. Let F be the magnitude of the horizontal force when the triangle just starts to tip over. The forces acting on the triangle when it is on the horizontal surface are shown in Figure 4-4.

Each force identified 1 point. [4]

Since the triangle is just starting to tip over and is not sliding, the conditions of static equilibrium still apply and, in addition, $f_s \le \mu F_N$. [1]

(Students should not be penalized for using "=" instead of "≤" in intermediate steps. Finding the upper limit of the range is a valid technique.)

The forces in the x-direction must sum to zero. $F = f_s \le \mu F_N$ [1]

The forces in the y-direction must sum to zero. $F_N = W$ [1]

The sum of the torques must also be zero. Taking torques about the lower left-hand corner,

$$0 = \sum \tau_i \quad [1]$$

Each torque can be written $\tau_i = R_i F_i \sin\theta_i$, or $\tau_i = R_{\perp i} F_i$ [1]

where $R_\perp$ is the perpendicular distance from the axis to line of action of the force.

$$0 = \sum \tau_i = FL - x_{cg} W \quad [2]$$

$$x_{cg} W = FL \le \mu WL \quad [1]$$

$$\mu \ge x_{cg} / L \quad [1]$$

$$\mu \ge 1/3 \quad [1]$$

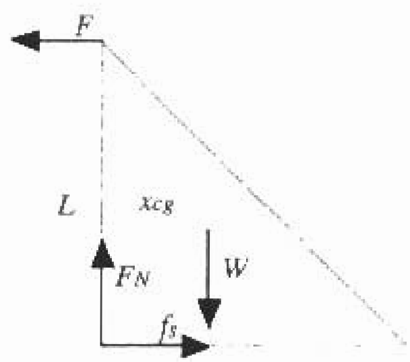


Figure 4-4

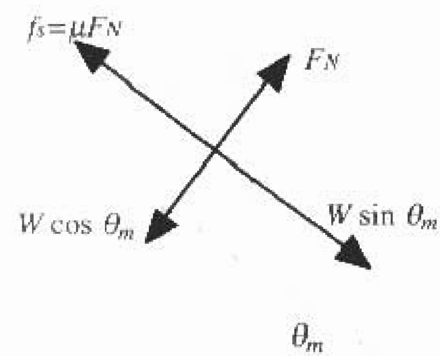


Figure 4-5

Let θ_m be the maximum inclination angle for which the triangle will not slide. The parallel and perpendicular components of the forces acting on the triangle are shown in Figure 4-5.

Each correct force component 1 point. [4]

The components perpendicular to the plane must sum to zero. $F_N = W\cos\theta_m$ [1]

The components parallel to the plane must sum to zero. $\mu F_N = W\sin\theta_m$. [1]

Combining the last two equations: $W\sin\theta_m = \mu W\cos\theta_m$

$$\tan\theta_m = \mu \ge x_{cg} / L \quad [1]$$

We can only be certain that θ_m is no smaller than $\theta_m = \tan^{-1}(x_{cg}/L)$ [1]

The range we can be certain of has θ_m as its maximum

$$\theta \le \tan^{-1}(x_{cg}/L) - \tan^{-1}(1/3) = 0.32 \text{ radians} = 18.4° \quad [3]$$

{Must have inequality for the full 3 points. Only one point for "=" instead of "≤".}